# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2021

## DSE4-MATHEMATICS

Full Marks: 60

**ASSIGNMENT**

*The figures in the margin indicate full marks.*
*All symbols are of usual significance.*

**The question paper contains DSE4A and DSE4B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.**

## DSE4A

## DIFFERENTIAL GEOMETRY

### GROUP-A

1. Answer ***all*** questions: 2×5 = 10

(a) If $r = r(s)$ be the position vector of a point $P$ with arc length $s$ as the parameter of the curve then show that $\tau = \dfrac{[r', r'', r''']}{|r''|^2}$.

(b) Find the torsion for the curve $r = (u^3 + 3u,\ 3u^2,\ u^3 - 3u)$.

(c) Show that a necessary and sufficient condition for a curve to be straight line is $\kappa = 0$.

(d) Find the envelope of the surface $3xt^2 - 3yt + z = t^3$.

(e) Find the lines of curvature on a plane.

### GROUP-B

2. Answer ***all*** questions: 10×3=30

(a) (i) Find the intrinsic equation of the curve $r = (ae^u \cos u,\ ae^u \sin u, be^u)$. 5+5

(ii) Show that the surface $e^z \cos x = \cos y$.

(b) (i) Show that the first fundamental form is invariant under a transformation of parameters. 5+5

(ii) Find the edge of regression of the family of planes $x \sin\theta - y\cos\theta + z = a\,\theta$, where $\theta$ is a parameter.

(c) (i) Discuss the nature of geodesics on a sphere. 5+5

(ii) Show that the curves $u+v=$ constant are geodesics on a surface with metric $ds^2=(1+u^2)\,du^2-2uv\,du\,dv+(1+v^2)\,dv^2$.

**GROUP-C**

3. Answer ***all*** questions: 5×2 = 10

(a) Show that the tangent to the locus of the centre of oscillating sphere passes through the centre of the osculating circle. 5+5

(b) If $R_s$ is the radius of spherical curvature, show that $R_s=\dfrac{|\hat{t}\times\hat{t}''|}{\kappa^2\tau}$.

**GROUP-D**

4. Answer ***all*** questions: 5×2 = 10

(a) If $L$, $M$, $N$ vanish at all points of a surface then the surface is plane, where $L$, $M$, $N$ are second fundamental coefficients. 5+5

(b) State and prove the Serret-Frenet formula in matrix form $\hat{e}_i'=\sum_{j=1}^{3}a_{ij}\hat{e}_j$, where the matrix $A=[a_{ij}]$ is Cartan matrix and $\hat{e}_1\equiv\hat{t}$, $\hat{e}_2\equiv\hat{n}$ and $\hat{e}_3\equiv\hat{b}$.

## DSE4B

## THEORY OF EQUATION

**GROUP-A**

1. Answer ***all*** the questions: 2×5 = 10

(a) If one of the roots of the equation $x^3+px^2+qx+r=0$ equals the sum of the other two, then proved that

$$p^3+8r=4pq$$

(b) Show that the equation of the form

$$\frac{x^4}{4!}+\frac{x^3}{3!}+\frac{x^2}{2!}+x+1=0$$

can not have a multiple root.

(c) If $\alpha$, $\beta$, $\gamma$ be the roots of the equation $x^3+px+q=0$, show that $\sum\alpha^5=5pq$.

(d) Show that $x^2-x+1$ is a factor of $x^{20}+x^{10}+1$.

(e) If $\alpha$ be an imaginary root of $x^{11}-1=0$, prove that $(\alpha+1)\,(\alpha^2+1)......(\alpha^{10}+1)=1$.

## GROUP-B

**Answer *all* the questions** 10×3=30

2. (a) Find the range of values of $r$ for which the equation $3x^4+8x^3-bx^2-24x+r=0$ has four real and unequal roots. 4+3+3

(b) Find the condition that the equation $x^4+px^3+qx^2+rx+s=0$ should have its roots $\alpha, \beta, \gamma, \delta$ connected by the relation $\alpha+\beta=0$.

(c) Solve the equation $x^3-x^2+3x-27=0$ having three distinct roots of equal moduli.

3. (a) Prove that the roots of the equation $x^3-6x-4=0$ are –2, $2\sqrt{2}\cos\frac{\pi}{12}$, $2\sqrt{2}\cos\frac{7\pi}{12}$. 4+4+2

(b) Show that the special roots of the equation $x^{10}-1=0$ are the non-real roots of the equation $x^5+1=0$.

(c) Is the equation $x^4-x^3+x^2+x-1=0$ a reciprocal equation? Justify your answer.

4. (a) Solve by Ferrari's method of the equation 4+4+2

$$2x^4+5x^3-8x^2-17x-6=0$$

(b) Prove that $(x^3+1)(x^2-x+1)=a(x^5+1)$ is a reciprocal equation if $a\neq 1$ and solve it when $a=2$.

(c) By Rolle's theorem, find the number and positions of the real roots of the equation $x^3-12x+7=0$.

## GROUP-C

5. Answer ***all*** the questions:

(a) The sum of two roots of the equation 5+5

$$x^4-8x^3+19x^2+4\lambda x+2=0$$

is equal to the sum of the other two. Find $\lambda$ and solve the equation.

(b) Use Sturm's theorem to show that the equation $x^4-3x^3-2x^2+7x+3=0$ has one root between –2 and –1, one root between –1 and 0 and two roots between 2 and 3.

## GROUP-D

6. (a) If $\alpha$, $\beta$, $\gamma$, $\delta$ be the roots of the biquadratic $x^4+px^3+qx^2+rx+s=0$, then find the equation whose roots are 5+5

$$(\beta\gamma+\alpha\delta),\ (\gamma\alpha+\beta\delta),\ (\alpha\beta+\gamma\delta)$$

Hence find the value of

$$(\alpha+\beta)(\alpha+\gamma)(\alpha+\delta)(\beta+\gamma)(\beta+\delta)(\gamma+\delta)$$

(b) Find the equation of the squared differences of the roots of the cubic $x^3+x^2-x=1$. Hence show that two roots of this equation are equal.

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