UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 6th Semester Examination, 2021

DSE3-MATHEMATICS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

The question paper contains DSE3A and DSE3B. Candidates are required to answer any *one* from the *two* courses and they should mention it clearly on the Answer Book.

DSE3A

POINT SET TOPOLOGY

GROUP-A

Answer *all* questions $2\times5 = 10$

1. (a) Show that sequences are continuous functions. 2

   (b) Show that $\mathbb{R}$ and $\mathbb{C}$ with their respective standard topologies cannot be homeomorphic. 2

   (c) The cofinite topology on a non-empty set X is the collection of subsets whose complements are either finite or all of X. Show that $\mathbb{R}$ with usual topology is not compact but $\mathbb{R}$ with cofinite topology is compact. 2

   (d) Find a condition (iff) on a given non-empty set X, so that it becomes compact. 2

   (e) Let $X = \{a, b, c, d\}$ be a topological space with the topology $Y = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $A = \{b, c\}$. Find derived set and interior of A. 2

GROUP-B

Answer *all* questions $10\times3 = 30$

2. (a) Show that every infinite set has an enumerable subset. 3

   (b) Let A be an enumerable set. Let $a \in A$ be fixed. Obtain the set $A' = A \setminus \{a\}$. Show that A and A' are equipotent. 3

   (c) Use above two results to prove that a set is infinite if and only if it admits a bijection with a proper subset of itself. 4

3. (a) Let $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ be continuous, consider the graph $G_\phi = \{(x, \phi(x))\,;\; x \in \mathbb{R}\}$ of ϕ as a subspace of $\mathbb{R}^2$. Show that G_ϕ is a homeomorphic copy of $\mathbb{R}$ embedded in $\mathbb{R}^2$. 4

(b) On the set of all positive integers $\mathbb{N}$, show that the metric d defined as $d(m, n)=\left|\frac{1}{m}-\frac{1}{n}\right|$, $m, n\in\mathbb{N}$ is equivalent to the discrete metric. Show that $\mathbb{N}$ is complete with respect to discrete metric, whereas it is incomplete with respect to d. 3+2+1

4. Let N denote the set of all null sequences of real numbers, that is $N=\{(x_n)_{n\in\mathbb{N}} : x_n \to 0\}$. Find closure $\overline{N}$ of N in $\mathbb{R}^{\omega}$ in both box and product topologies, where $\mathbb{R}^{\omega}$ denotes the product of countable copies of $\mathbb{R}$. 5+5

GROUP-C

Answer *all* questions 5×2 = 10

5. A topological space is called a Hausdorff space if any two distinct points in the space can be separated by two disjoint open sets. Show that a topological space X is Hausdorff if and only if the diagonal $\Delta=\{(x, x): x\in X\}$ is closed in $X\times X$. 5

6. Let $p: X\to Y$ be a closed, continuous and surjective map such that for every point $y\in Y$, $p^{-1}\{y\}$ is compact in X. Show that if Y is compact, then X is compact. 5

GROUP-D

Answer *all* questions 5×2 = 10

7. (a) Investigate the convergence and the possible limit(s) of the sequence $\left\{x_n=\frac{1}{n}\right\}$ in the cofinite topology on $\mathbb{R}$. 2

(b) Show that a topological space is connected if and only if every non-empty proper subset has a nonempty boundary. 3

8. Let X be a connected topological space and $f: X\longrightarrow\mathbb{R}$ is a non-constant continuous map. Show that X is an uncountable set. 5

DSE3B

BOOLEAN ALGEBRA AND AUTOMATA THEORY

GROUP-A

Answer *all* questions 2×5 = 10

1. (a) What is the language generated by the Grammar $(\{S\}, \{a, b\}, \{S\longrightarrow aS, S\longrightarrow bS, S\longrightarrow\epsilon\}, S)$?

(b) Determine all the sub-lattices of D_{30} that contains at least four elements.

(c) Draw the logic circuit $(A'B)'+(A+C)'$.

(d) Show that the weak distributive law $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$ holds for any lattice L.

(e) Prove that $L = \{a^n b^n c^n \mid n \geq 1\}$ is not a context free language?

GROUP-B

10×3 = 30

2. (a) For the Grammar = $\{V, T, P, S\}$, where $S \longrightarrow 0B$, $A \longrightarrow 1AA|\epsilon$, $B \longrightarrow 0AA$, construct a parse tree. 4+3+3

(b) Convert the given NFA to equivalent DFA.

δ	0	1
$\longrightarrow p$	$\{p, q\}$	$\{p\}$
q	$\emptyset$	$\{r\}$
r^*	$\{p, r\}$	$\{q\}$

(c) Design a PDA for recognizing the language of palindromes over the alphabet $\{0, 1\}$. Draw the computation tree showing all possible moves for the strings 00100 and 00101.

3. (a) Let E and F be finite ordered sets. If $f : E \longrightarrow F$ is a bijection, prove that f is an order isomorphism if and only if $(\forall a, b \in L)$ $x \prec y \Leftrightarrow f(x) \prec f(y)$, where $x \prec y$ means 'y covers x'. 3+2+3+2

(b) In a distributive lattice $(A, \leq)$, if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a then show that $x = y$.

(c) Suppose P be an ordered set with the property: for any $x, y \in P$, $x \wedge y = \text{g.l.b.}(x, y)$ and $x \vee y = \text{l.u.b.}(x, y)$. Prove that $(P, \wedge, \vee)$ is a lattice.

(d) Show that for any elements a, b, c in a modular lattice,

$$(a \vee b) \wedge c = b \wedge c \text{ implies } (c \vee b) \wedge a = b \vee a.$$

4. (a) Using the laws of Boolean Algebra, show that 3+3+4

$$[x'.(x+y)]'+[y.(y+x')]'+[y'.(y'+x)]'=1$$

(b) Let $E = xy' + xyz' + x'yz'$. Prove that (i) $xz' + E = E$, (ii) $x + E \neq E$.

(c) Draw the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit.

x	y	z	$f(x, y, z)$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

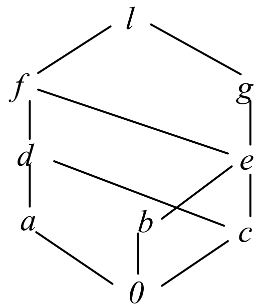
GROUP-C

10×1 = 10

5. (a) Design a DFA that accepts the following languages: 5

$L_1 = \{x \in \{0, 1\}^* : x \text{ ends in } 00\}$ and $L_2 = \{x \in \{0, 1\}^* : x \text{ contains three consecutive } 0's\}$.

(b) Consider the bounded lattice L in the following figure: 5



(i) Find the complements, if they exist, of e and f.

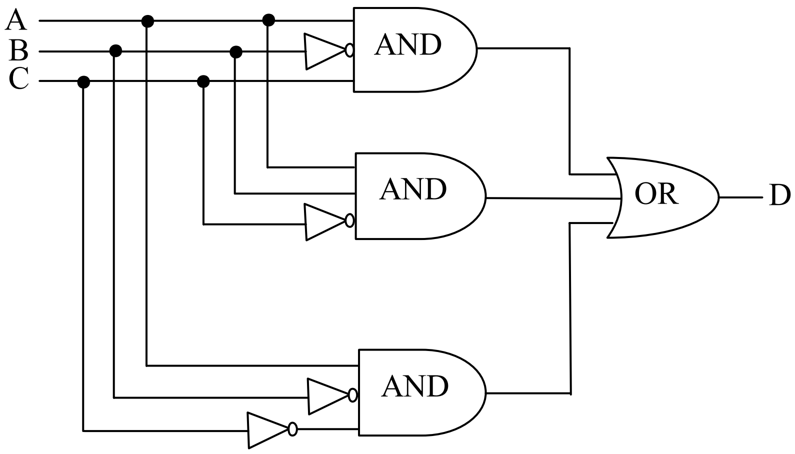
(ii) Is L distributive?

(iii) Describe the isomorphisms of L with itself.

GROUP-D

10×1 = 10

6. (a) Use Karnaugh maps to redesign the following logic circuit so that it becomes a minimal AND-OR Circuit. 6+4



(b) For $\sum = \{a, b\}$, design a Turing machine that accepts $L = \{a^n b^n : n \geq 1\}$. Compute an ID for the string *aabb*.

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