UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2021

CC4-MATHEMATICS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** the questions: 2×5=10

(a) Calculate $\lim\limits_{t\to 3}\vec{r}(t)$, where $\vec{r}(t)=\left(\frac{2t-4}{t+1}\right)\hat{i}+\left(\frac{t}{t^2+1}\right)\hat{j}+(4t-3)\hat{k}$.

(b) Examine whether the vector valued function $\vec{r}(t)=t^2\hat{i}+e^t\hat{j}+\frac{1}{t+3}\hat{k}$ is continuous at $t=-3$ or not.

(c) Find the angle between the normals to the following surfaces $y^2+z^2=9$ and $2(x^2-z^2)=3y$ at the point (2, 2, 1).

(d) Show that the integral $\int_C y\,dx+x\,dy$ is independent of the path C joining the points $P(0, 1)$ and $Q(1, 2)$.

(e) Find the particular integral of the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=xe^{-x}$.

GROUP-B

2. Answer ***all*** the questions: 10×3=30

(a) (i) Solve the differential equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+4y=e^x\cos x$. 5+5=10

(ii) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$.

(b) (i) Solve the Euler's equation $x^2\frac{d^2y}{dx^2}-9x\frac{dy}{dx}+25y=0$. 5+5=10

(ii) Solve: $\frac{dx}{dt}+\frac{dy}{dt}-2y=2\cos t-7\sin t$

$\frac{dx}{dt}-\frac{dy}{dt}+2x=4\cos t-3\sin t$

(c) (i) Evaluate the integral of $\vec{F} = (yz + zx)\vec{i} + xz\,\vec{j} + (xy + 2z)\vec{k}$ along the circle $x^2 + y^2 = 1,\ z = 1$ from (0, 1, 1) to (1, 0, 1). 5+5=10

(ii) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = (x^2 - 3y^2)\vec{i} + (y^2 - 2x^2)\vec{j}$ s and C the closed curve in xy plane given by $x = 3\cos t,\ y = 2\sin t,\ 0 \le t \le 2\pi$, C is described in the anti-clockwise sense.

GROUP-C

3. Answer ***all*** the questions: 5×2=10

(a) Solve $(D^2 - 2D + 4)y = (x + x^3)e^{2x}$ by method of undetermined coefficient.

(b) Apply Picard's method up to third approximation to solve

$$\frac{dy}{dx} = 3e^x + 2y \ ; \quad y(0) = 0.$$

GROUP-D

4. Answer ***all*** the questions: 5×2=10

(a) Show that equation of the tangent line to the curve $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$ at the point $t = 1$ is $2(x-1) = (y-1) = z - \frac{2}{3}$.

(b) Solve: $\frac{dx}{dt} + 2x - 3y = t$

$$\frac{dy}{dt} - 3x + 2y = e^{2t}$$

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