



UNIVERSITY OF NORTH BENGAL

B.Sc. Honours Part-II Examination, 2021

MATHEMATICS

PAPER-VIII

Full Marks: 50

ASSIGNMENT

*The figures in the margin indicate full marks.
All symbols are of usual significance.*

GROUP-A

1. Answer *all* questions: 5×5 = 25

(a) Show that the locus of the point from which three mutually perpendicular lines can be drawn to intersect the conic 5
 $z = 0, ax^2 + by^2 = 1$ is $ax^2 + by^2 + (a+b)z^2 = 1$

(b) (i) Obtain the equation of the sphere through the points (1, 1, 2) and (2, -2, 3) and having its centre on the line $2x + 3y = 0 = 5x + y - z$. 2+3

(ii) A variable plane which is at a constant distance $3p$ from the origin O cuts the axes in A, B, C . Show that the locus of the centroid of the triangle ABC is

$$x^{-2} + y^{-2} + z^{-2} = p^{-2}$$

(c) (i) Find the angle between the lines whose direction cosines satisfy the equations 2+3
 $l + m + n = 0$ and $2nl + 2lm - mn = 0$

(ii) Show that the straight line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ meets the locus of the equation $ax^2 + by^2 + cz^2 = 1$ in two points.

(d) Find the locus of luminous points of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ that casts a circular shadow on the plane $z = 0$. 5

(e) A variable plane is parallel to the given plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C . Prove that the circle ABC lies on the cone 5

$$yz\left(\frac{b}{c} + \frac{c}{a}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

GROUP-B

2. Answer *all* questions: 5×5 = 25

(a) (i) Using Laplace transform, solve the initial value problem: 4+1

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sin x, \quad \text{given that } y(0) = 0, \quad y'(0) = 1$$

(ii) Form a partial differential equation by eliminating the function ϕ from

$$lx + my + nz = \phi(x^2 + y^2 + z^2)$$

(b) Find the series solution of $x^2y'' + xy' + (x^2 - 4)y = 0$ about $x = 0$. 5

(c) Solve: $y'' - 4xy' + (4x^2 - 1)y = -3e^{x^2} \sin 2x$ by reduction to Normal form. 5

(d) Find a complete integral of $px^2 + 2qxy - pq = 2xz$ by Charpit's method, where symbols have their usual meaning. 5

(e) Find the eigen values and the corresponding eigen functions for the given differential equation. 5

$$\frac{d^2y}{dx^2} + \lambda y = 0, \quad \lambda > 0 \quad \text{given that } y(0) + y'(0) = 0, \quad y(1) + y'(1) = 0$$

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