# UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2020

## DSE2-MATHEMATICS

Full Marks: 60

### ASSIGNMENT

*The figures in the margin indicate full marks.*
*All symbols are of usual significance.*

**The question paper contains DSE2A and DSE2B. Candidates are required to answer any *one* from the *two* DSE2 courses and they should mention it clearly on the Answer Book.**

## DSE2A

## NUMBER THEORY

### GROUP-A

1. Answer ***all*** questions: $2\times6 = 12$

(a) List all prime numbers that divides 65!. 2

(b) Prove that the product of any three consecutive integers is divisible by 6. 2

(c) Find the last digit of $4^{4^4}$. 2

(d) Show that $\gcd(a,\ a+2)=1$ or 2 for every integer $a$. 2

(e) Find the integers $u$ and $v$ satisfying $52u-91v=78$. 2

(f) Prove that $\sqrt{2}$ is an irrational number. 2

### GROUP-B

**Answer *all* questions** $5\times4 = 20$

2. Solve the system of linear congruences: 5

$$x\equiv 2 \pmod 5$$
$$x\equiv 3 \pmod 7$$
$$x\equiv 5 \pmod 8$$

3. Use Euclidean Algorithm to find gcd(1769, 2378). 5

4. Find an inverse of 13 modulo 1000. 5

5. Find the least positive residue $r$ such that $2^{41}\equiv r \pmod{23}$. 5

## GROUP-C

**Answer *all* questions** 7×4 = 28

6. (a) Find the remainder when $1! + 2! + 3! + \cdots + 50!$ is divided by 15. 4+3

   (b) Prove that no prime factor of $n^2 + 1$ can be of the form $4m - 1$.

7. Find the general solution in integers and positive integral solutions of the equation 4+3

$$172x + 20y = 1000$$

8. (a) Find the unit digit in $77^{77}$. 4+3

   (b) If $\gcd(a, 133) = \gcd(b, 133) = 1$, then prove that $a^{18} - b^{18}$ is divisible by 133.

9. (a) Express 100 as a sum of two integers so that one of them is divisible by 7 and the other is divisible by 11. 3+4

   (b) If $p$ be an odd prime, prove that

$$1^2 \cdot 3^2 \cdot 5^2 \cdots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod p$$

# DSE2B

# MECHANICS

## GROUP-A

1. Answer ***all*** the questions: 2×6 = 12

   (a) What do you mean by a system is in astatic equilibrium under a system of Coplanar Forces? 2

   (b) Show that every system of forces acting on a rigid body can be reduced to a wrench. 2

   (c) If a particle moves in a circle of radius $r$ with uniform speed $v$, then prove that its angular velocity about the centre is constant and equal to $v/r$. 2

   (d) Three forces $P$, $Q$, $R$ act along the sides of the triangle formed by the line 2

$$x + y = 1, \quad y - x = 1, \quad y = 2$$

   Obtain the line of action of their resultant.

   (e) If the curve $\frac{2a^3}{r^3} = 1 + \cos 3\theta$ be described by a central force, then show that the force, directed to the pole, is constant. 2

   (f) Define stable and unstable equilibrium. 2

## GROUP-B

2. Answer ***all*** the questions: 5×4 = 20

   (a) Deduce the condition of stability of an orbit which is nearly circular under the action of a central force $F = \varphi(u)$, where $u = 1/r$. 5

(b) A uniform straight rod rests in a vertical plane with one end resting against a rough vertical wall and the lower end on a rough horizontal plane. If the friction is limiting at both ends when the inclination to the horizontal is $\alpha$, and the coefficient of friction is the same for both contacts, prove that the angle of friction is $\frac{\pi}{4}-\frac{\alpha}{2}$. 5

(c) If the resistance of the air to a particle motion be $n$ times its weight, and the particle be projected horizontally with velocity $V$, show that the velocity of the particle, when it is moving at an inclination $\varphi$ to the horizontal is $V(1-\sin\varphi)^{\frac{n-1}{2}}(1+\sin\varphi)^{-\frac{n+1}{2}}$. 5

(d) Three forces act along the straight lines $x=0$, $y-z=a$; $y=0$, $z-x=a$; $z=0$, $x-y=a$. Show that if the system reduces to a single force its line of action must lie in the surface $x^2+y^2+z^2-2yz-2zx-2xy=a^2$. 5

## GROUP-C

3. Answer ***all*** the questions: $7\times4=28$

(a) (i) A sphere of weight $W$ and radius $a$ lies within a fixed spherical shell of radius $b$, and a particle of weight $\omega$ is fixed to the upper end of the vertical diameter, prove, that the equilibrium is stable if $\frac{W}{\omega}=\left(\frac{b-2a}{a}\right)$. 3+4

(ii) Two forces $2P$ and $P$ act along the lines whose equations are $y=x\tan\alpha$, $z=c$ and $y=-x\tan\alpha$, $z=-c$ respectively. Prove that the equation of the central axis is $y=\frac{1}{3}(x\tan\alpha)$, $z=\frac{3c}{\sin^2\alpha+9\cos^2\alpha}$.

(b) (i) An artificial satellite is circling round the earth with the same centre as the centre of the earth. Show that, $\frac{v_s}{v_0}=\sqrt{2}$, where $v_s$, $v_0$ are respectively the escape velocity and the orbital velocity of the satellite. 5+2

(ii) Write down the Kepler's law of planetary motion.

(c) (i) One end of an elastic string of unstretched length $a$, is tied to a point on the top of a smooth table, and a particle attached to the other end can move freely on the table. If the path be nearly circular of radius, show that its apsidal angle is approximate, $\pi\sqrt{\frac{b-a}{4b-3a}}$. 4+3

(ii) A hemispherical shell on a rough plane, whose angle of friction is $\lambda$, show that the inclination of the plane base of the rim to the horizontal cannot be greater than $\sin^{-1}(2\sin\lambda)$.

(d) (i) If $T$ be the time period of a satellite circling round the earth at a distance $r$ from the earth center, then prove that $r=\left(\frac{g_0R^2T^2}{4\pi^2}\right)^{1/3}$, where $g_0$ is the acceleration due to gravity on earth's and $R$ is the radius of the earth. 4+3

(ii) The density at any point of a circular lamina varies as the $n^{\text{th}}$ power of the distance from a point $O$ on the circumference. Show that c.g. of the lamina divides the diameter through $O$ in the ratio $(n+2):2$.

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