UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2020

DSE1-MATHEMATICS

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

The question paper contains DSE1A and DSE1B. Candidates are required to answer any *one* from the *two* DSE1 courses and they should mention it clearly on the Answer Book.

DSE1A

PROBABILITY AND STATISTICS

GROUP-A

1. Answer ***all*** questions: 2×6=12

(a) If A and B are two events and $P(A\cup B)=\frac{1}{2}$, $P(A)=\frac{2}{5}$, $P(B)=\frac{1}{4}$, find the value of $P(A'\cup B')$. 2

(b) Using Tchebycheff's inequality, show that for a random variable X having p.d.f. 2

$$f(x) = 1 \;,\quad \text{if} \quad 0<x<1$$
$$\quad = 0 \;,\quad \text{otherwise}$$

$P\left\{\left| X-\frac{1}{2}\right| \le 2\cdot\frac{1}{\sqrt{12}}\right\} \ge \frac{3}{4}$.

(c) Give an example to contradict the expression $E(XY)=E(X)\cdot E(Y)$. 2

(d) Find the probability of getting a total of 7 atleast once in three tosses of a pair of fair dice. 2

(e) If X be a Poisson random variable and $E(x)=\lambda$, then find $E\{(x+1)^2\}$. 2

(f) If a random variable X has p.d.f. 2

$$f(x)=\frac{1}{2}e^{-|x|} \;,\quad -\infty<x<\infty.$$

Show that the moment generating function $M_x(t)=\dfrac{1}{1-t^2}$.

GROUP-B

Answer *all* questions 5×4=20

2. If X is Normal (m,σ), then prove that 5

$$P(a<X<b)=\phi\left(\frac{b-m}{\sigma}\right)-\phi\left(\frac{a-m}{\sigma}\right) \quad \text{and} \quad P(|X-m|>a\sigma)=2\{1-\phi(a)\},$$

where ϕ is the standard normal distribution function.

3. (a) A point P is choosen at random on a circle of radius a and A is a fixed point on the circle. Find the probability that the chord AP will exceed the length of an equilateral triangle inscribed in the circle. 3

(b) In a collection of 6 Mathematics books and 4 Physics books, find the probability that 3 particular Mathematics books will be together. 2

4. A continuous random variable X has the probability density function 5

$$f(x) = \frac{1}{2} - \frac{x}{8} \quad , \quad (0 \le x \le 4)$$

Find the cumulative distribution function and calculate the probabilities

(i) $P(X \le 1)$

(ii) $P(X \ge 2.5)$

(iii) $P(|X-2| < 0.5)$

5. (a) Give an interpretation of weak law of large numbers for the appearances of a 3 in successive tosses of a fair die. 4+1

(b) If X is a positive random variable with mean 3, then find $P(|X-3|<1)$.

GROUP-C

Answer *all* questions 7×4=28

6. (a) From a pack of 52 cards an even number of cards are drawn. Show that the probability that these consist of half red and half black is 4+3

$$\left\{\frac{52\,!}{(26!)^2} - 1\right\} \Big/ (2^{51} - 1)$$

(b) From a point on a circle of radius r, chords are drawn at random. Find mean and variance of the length of chords.

7. (a) Find the characteristic function $\chi_x(t)$ of a random variable having distribution with density 3+4

$$\begin{aligned} f(x) &= 1 - |x| \quad , \quad \text{if} \quad |x| < 1 \\ &= \quad 0 \qquad , \quad \text{elsewhere.} \end{aligned}$$

(b) Can the function

$$f(x) = \begin{cases} C(1-x^2) & , \quad 0 \le x \le 1 \\ \quad 0 & , \quad \text{otherwise} \end{cases}$$

be a distribution function? — Explain.

8. (a) Let the random variable X_i takes up values i and $-i$ with equal probabilities, $i = 1, 2, \cdots, n$. Show that the law of large numbers cannot be applied to the mutually independent variables $X_1, X_2, \cdots\cdots, X_n$. 4+3

(b) The distribution of a random variable X is given by $P(X=-1) = \frac{1}{8}$, $P(X=0) = \frac{3}{4}$, $P(X=1) = \frac{1}{8}$. Verify Tchebycheff's inequality for the distribution.

9. (a) If X_1 and X_2 are independent and normally distributed with mean 0 and variance 1. Then find the distribution for $Y = \frac{X_1}{X_2}$. 4+3

(b) If x be a binomially distributed random variables with parameter n and p therefore what values of p, var (x) will be maximum, assuming n is fixed.

DSE1B

LPP

GROUP-A

1. Answer ***all*** questions: 2×6=12

(a) Find the extreme points of the set 2

$$S = \{(x_1,\ x_2)\ :\ -x_1 + x_2 = 4,\quad x_1 \geq 0,\ \ x_2 \geq 0\}$$

(b) Transform the following pay-off matrix of a zero sum game into a linear programming problem for player B. 2

	B_1	B_2	B_3
A_1	9	1	4
A_2	0	6	3
A_3	5	2	8

(c) Express the point (0.4, 0.6) as a convex combination of the points (0, 0), (1, 0), (1, 1), (0, 1). 2

(d) What is the convex hull of the set $X = \left\{(x,\ y)\ \middle|\ \frac{x^2}{3} + \frac{y^2}{2} = 1\right\}$? 2

(e) Find all possible basic solutions of the following set of equations 2

$$x_1 + 2x_2 - x_3 = 5$$

$$-x_1 + 3x_2 + x_3 = 5$$

(f) Show that the vectors (1, 0, 0), (0, 1, 1) and (0, 0, 1) form a basis of the space E^3. 2

GROUP-B

Answer *all* questions 5×4=20

2. In a hospital, meals are served twice to patients. Each gram of the first meal contains 10 units of protein and 20 units of vitamins and each gram of second meal contains 15 units of protein and 15 units of vitamins. In a day's meal each patient should get atleast 50 units of vitamins and 60 units of protein. Cost of each gram of first meal is Rs. 6 and that of the second is Rs. 5. Formulate an L.P.P. as to minimize the daily cost of food. Hence solve it. 5

3. Solve graphically the following LPP: 5

$$\text{Min}\ \ z = x_1 + x_2$$

Subject to

$$5x_1 + 9x_2 \leq 45$$

$$x_1 + x_2 \geq 2$$

$$x_1 \leq 4$$

$$x_1,\ \ x_2 \geq 0$$

4. Use two phase method to show that the following LPP has unbounded solution: 5

Max $z = 2x_1 + 3x_2 + x_3$, $x_1,\ x_2,\ x_3 \geq 0$

Subject to $-3x_1 + 2x_2 + 3x_3 = 8$

$-3x_1 + 4x_2 + 2x_3 = 7$

5. Solve the following game problem graphically: 5

Player *B*	3	2	–1	4	3	4
Player *A*	2	5	6	–2	4	2

GROUP-C

Answer *all* questions 7×4=28

6. Solve the following LPP by Simplex method: 7

Maximize $z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 8$

$x_1 + 2x_2 = 5$

$2x_1 + x_2 \leq 8$

$x_1 \geq 0,\ \ x_2 \geq 0$

7. Using principle of dominance, solve the following game: 7

Player *B*

$$\text{Player } A \quad \begin{pmatrix} 1 & 7 & 2 \\ 0 & 2 & 7 \\ 5 & 1 & 6 \end{pmatrix}$$

8. (a) Reduce the feasible solutions $x_1 = 3,\ x_2 = 1,\ x_3 = 0,\ x_4 = 3$ to basic feasible solutions of the following set of equations: 4+3

$4x_1 + x_2 - x_3 + x_4 = 16$

$x_1 + 2x_2 + 3x_3 + 2x_4 = 11$

(b) Solve the following LPP:

Maximize $z = -3x_1 + 2x_2$

Subject to $-x_1 + 4x_2 \geq 14$

$-3x_1 + 2x_2 \leq 6$

$x_1,\ \ x_2 \geq 0$

9. (a) $x_1 = 3,\ x_2 = 1,\ x_3 = 1$ and $x_4 = 2$ is a feasible solution of the set of equations: 5+2

$x_1 + x_2 + x_3 + x_4 = 7$

$2x_1 + x_2 + 2x_3 + x_4 = 11$

$x_2 + x_3 + 2x_4 = 6$

Reduce the feasible solution to one or more basic feasible solutions.

(b) Show that the intersection of a convex set is a convex set.

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