UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 5th Semester Examination, 2020

CC11-MATHEMATICS

GROUP THEORY-II

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer the following questions: $2\times6 = 12$

(a) Let G be a group and $a \in G$. Show that $\langle a \rangle = \{a^n \mid n \in \mathbb{Z}\}$. 2

(b) Show that $4\mathbb{Z}\big| 12\,\mathbb{Z} \simeq \mathbb{Z}_3$. 2

(c) Let G be a group and $x \in G$. Define stabilizer of x in G. 2

(d) State the class equation for a finite group. 2

(e) Let $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ be defined by $f(x) = x^2$. Show that f is a homomorphism from $(\mathbb{R}^+, \cdot)$ to $(\mathbb{R}^+, \cdot)$. Find the kernel of f. 2

(f) Define an action of a group on a set X. 2

GROUP-B

2. Answer the following questions: $5\times4 = 20$

(a) If G acts on X and $x \in X$, then show that $\text{stab}_G(x)$ is a subgroup of G. 5

(b) If G is a finite group of order p^2, where p is prime, then show that G is abelian. 5

(c) Let G be a group, for $g \in G$ define $T_g : G \to G$ by $T_g x = g^{-1}xg$, $\forall\, x \in G$. Show that T_g is an automorphism. 5

(d) If G is a group of order $p^n (n > 0)$, where p is a prime, then prove that $Z(G) \neq \{e\}$. 5

GROUP-C

3. Answer the following questions: $7\times4 = 28$

(a) (i) Show that the number of even permutations in S_n, $(n \geq 2)$ is the same as that of the odd permutations. 5+2=7

(ii) Let G be a group. Define $f: G \to G$ by $f(a) = a^{-1}$, $\forall a \in G$. Prove that f is a homomorphism if G is commutative.

(b) (i) Let G be a group. Let us define a set $X = \{aba^{-1}b^{-1} : a, b \in G\}$. Show that X is a subgroup of G. 3+4=7

(ii) Let G be a group. Show that $\mathscr{I}(G)$ is a normal subgroup of Aut(G), where $\mathscr{I}(G)$ is the inner automorphism of G.

(c) Show that the number of conjugate classes in S_n is $p(n)$, the number of partitions of n. 7

(d) (i) Let G be a group and φ an automorphism of G. If $a \in G$ is of order $O(a)$, then show that $O(\varphi(a)) = O(a)$. 3+4=7

(ii) Let G be a group. Show that $\mathscr{I}(G) \simeq G \big| Z(G)$, where $\mathscr{I}(G)$ is the group of inner automorphisms.

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