UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 1st Semester Examination, 2020

CC1-MATHEMATICS

CALCULUS, GEOMETRY AND DIFFERENTIAL EQUATION

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

GROUP-A

1. Answer ***all*** the questions: 2×6 = 12

(a) Find all the intervals on which function f given by $f(x) = cmx$ is concave upwards.

(b) Show that the α-discriminant of the semicubical parabola $(x+a)^2 + y^3 = 0$ is a cusp locus.

(c) Find k so that the equation $kx^2 + 4xy + y^2 - 6x - 2y + 2 = 0$ may represent a point ellipse.

(d) Solve: $1 + y^2 + (x - e^{-\tan^{-1} y})\dfrac{dy}{dx} = 0$

(e) The axes are rotated through an angle 45° without changing the origin. Find the form of the equation $x^2 - y^2 = a^2$ in the new system.

(f) If $I_n = \int_0^1 x^n e^{-x^2/2}\,dx$, $n \in \mathbb{N}$. Prove that, $I_n = (n-1)I_{n-2} - e^{-1/2}$, $n \geq 2$.

GROUP-B

Answer *all* questions 5×4 = 20

2. (a) The distance x described by a person pulling by means of a parachute satisfies the differential equation 2+3

$$\left(\frac{dx}{dt}\right)^2 = k^2\left(1 - e^{-\frac{2gx}{k^2}}\right)$$

where k and g are constant and $x = 0$ when $t = 0$. Show that $x = \dfrac{k^2}{g}\cosh\left(\dfrac{gt}{k}\right)$.

(b) Solve: $\sec^2 y\,\dfrac{dy}{dx} + 2x\tan y = x^3$

3. (a) If $y = e^{\cos^{-1} x}$, show that an equation connecting y_n, y_{n+1} and y_{n+2} is given by $2\frac{1}{2}+2\frac{1}{2}$

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+1)y_n = 0$$

(b) Find the area of the region included between the following curve and its asymptotes:

$$x^2y^2 = a^2(y^2 - x^2)$$

4. (a) Show that the equation $7x^2 - 2xy + 7y^2 + 22x - 10y + 7 = 0$ represents an ellipse. Find its centre, the equations of the axes and the directrices. 2+3

(b) If $I_{m,\,n} = \int \cos^m x \cos nx \; dx$,

prove that, $I_{m,\,n} = \dfrac{\cos^m x \sin nx}{m+n} + \dfrac{m}{m+n} I_{m-1,\,n-1}$

$$= \frac{n\sin nx\,\cos x - m\cos nx\,\sin x}{n^2 - m^2}\cos^{m-1} x - \frac{m(m-1)}{n^2 - m^2}\,I_{m-2\,,\,n}$$

5. (a) Find the asymptotes of $x(x-y)^2 - 3(x^2 - y^2) + 8y = 0$. 2+3

(b) Find the length of perimeter of the cardioid $r = a(1+\cos\theta)$ and show that the arc of the upper half of the curve is bisected at $\theta = \pi/3$.

GROUP-C

Answer *all* questions 7×4= 28

6. (a) Trace the curve: $y^2(a+x) = x^2(3a - x)$ 4+3

(b) Solve: $(4x^2y - 6)\,dx + x^3dy = 0$ by putting $xy = z$

7. (a) Show that the spheres which cut two given spheres along great circles all pass through two fixed points. 4+3

(b) Reduce the equation $x^2 - 4xy + 4y^2 + 2x - 4y + c = 0$ to its canonical form and determine the type of the conic represented by it for different values of *c*.

8. (a) Find the surface of the solid generated by revolution of the astroid $x = a\cos^3 t$, $y = b\sin^3 t$ about x-axis. 4+2+1

(b) The smaller of the two arcs into which the parabola $y^2 = 8x$ divides the circle $x^2 + y^2 = 9$ is rotated about the y-axis. Find the volume of the solid thus generated.

(c) If $I_{m,\,n} = \int a^m (\ln x)^n \, dx$, show that $(m+1)I_{m,\,n} = x^{m+1}(\ln x)^n - n\,I_{m,\,n-1}$.

9. (a) Show that the envelop of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are variable parameters connecting by the relation $a+b=c$, c being a non zero constant, is $\sqrt{x} + \sqrt{y} = \sqrt{c}$. $3\frac{1}{2}+3\frac{1}{2}$

(b) Solve by the method of variation of parameters

$$\frac{dy}{dx} + e^t y = e^t$$

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